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J.-R. Chazottes and B. Fernandez (eds): Dynamics of Coupled Map Lattices and of Related Spatially Extended Systems

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The book presents a collection of lecture notes written for the Coupled Map Lattices (CML) summer school/conference held at the Institute Henri Poincaré in Paris in the beginning of the summer 2004.

By a CML one usually understands a finite or countable collection of 'sites', a simple, usually low-dimensional, dynamical system placed at each site and some 'coupling' to systems at other sites. The sites are often located at points in a regular lattice (whence the name) but this need not be so. A series of 15 lectures gives the reader an overview of the state of art in the subject. Instead of reviewing each lecture let me give a list of some of the important questions discussed in the book (more or less in this order):

1. Statistical properties of CML: The existence and uniqueness of a 'natural' or SRB measure, decay of correlations for invariant measures (hyperbolicity + weak-coupling). Counter-examples to unique SRB measures. Phase transitions.
2. Geometric aspects of CML: Pattern-formation. Synchronization versus 'chaos'. Topological properties and bifurcations.
3. The Frenkel–Kontorova model: Transport. Monotonicity and front propagation.
4. Dynamics of networks: DNA regulation of proteins. Coupled neuron network (dynamical point of view, not neural networks).

For several decades, CMLs have been at the center of the study of 'spatially extended systems'. From the first descriptions in the physics literature it has developed into a respectable, though difficult, part of the mathematical theory of dynamical systems. The present book puts the reader at the forefront of this theory. At present it seems that one needs the following ingredients: Either strong hyperbolicity (strong chaos locally) + weak coupling or strong dissipation (regular local behavior) + moderate, possibly strong, coupling. Still, many interesting phenomenae fall into these categories and await the attention of researchers in the field.

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